

# Hypothesis Testing with Error Correction Models

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## Abstract

Grant and Lebo (2016) and Keele, Linn, and Webb (2016) clarify the conditions under which the popular general error correction model (GECM) can be used and interpreted easily: In a bivariate GECM the data must be integrated in order to rely on the error correction coefficient,  $\alpha_1^*$ , to test cointegration and measure the rate of error correction between a single exogenous  $x$  and a dependent variable,  $y$ . Here we demonstrate that even if the data are all integrated, the test on  $\alpha_1^*$  is misunderstood when there is more than a single independent variable. The null hypothesis is that there is no cointegration between  $y$  and any  $x$  but the correct alternative hypothesis is that  $y$  is cointegrated with at least one—but not necessarily more than one—of the  $x$ 's. A significant  $\alpha_1^*$  can occur when some  $I(1)$  regressors are not cointegrated and the equation is not balanced. Thus, the correct limiting distributions of the right-hand-side long-run coefficients may be unknown. We use simulations to demonstrate the problem and then discuss implications for applied examples.

## 1 Introduction

A recent *Political Analysis* symposium investigated applications of the popular general error correction method (GECM). [Grant and Lebo \(2016\)](#) focus on common mistakes made with the GECM, particularly with interpreting the error correction parameter. A response by [Keele, Linn, and Webb \(2016\)](#) clarifies the meaning of often misunderstood parts of [DeBoef and Keele \(2008\)](#). The symposium sparked interest in both the usage of the method and the question of equation balance (e.g. [Enns and Wlezien 2017](#); [Lebo and Kraft 2017](#); [Pickup and Kellstedt 2018](#)). Here, we demonstrate interpretation problems when using multiple exogenous variables, even when all are unit roots. In particular, we outline the correct interpretation of the hypothesis test on the error correction coefficient,  $\alpha_1^*$ . Rejecting the null does *not* indicate that *all* of the variables are cointegrated. Further,  $\alpha_1^*$  cannot assess which  $x$ 's are cointegrated with  $y$ , whether the equation is balanced, or what the correct critical values are for other coefficients. We highlight the implications for applied research with examples of [Kelly and Enns's \(2010\)](#) and [Volscho and Kelly's \(2012\)](#) tests for cointegration.

## 2 Cointegration, the GECM, and Equation Balance

A simplified expression of an individual time series,  $y_t$ , is:

$$y_t = D_t + \rho y_{t-1} + \mu_t \tag{1}$$

in which the deterministic features – a constant or trend – are captured by  $D_t$  and  $\mu_t$  is a white noise process. When  $\rho = 1$ , the series has a unit root (non-stationary, integrated, or  $I(1)$ ) and meanders without tending towards a long-term mean. A series with  $\rho < 1$  will have mean reversion and is classified as stationary, non-integrated, or  $I(0)$ . A unit root series can be rendered stationary through the process of differencing – creating a new series from the changes between time-points, i.e.  $\Delta y_t = y_t - y_{t-1}$ .

*Cointegration* exists when a linear combination of two or more unit-root series are jointly stationary. Cointegrated series are in a long-run equilibrium such that any movement away from each other is short-lived. [Engle and Granger \(1987\)](#) provide a two-step framework for understanding and testing for cointegration that begins with:

$$y_t = \alpha + \beta x_t + \varepsilon_t, \tag{2}$$

where  $y_t$  and  $x_t$  are both  $I(1)$  and the residuals are  $\hat{\varepsilon}_t$ .<sup>1</sup> Testing  $\hat{\varepsilon}_t$ 's stationarity is a cointegration test. If  $\hat{\varepsilon}_t$  is stationary,  $\hat{\varepsilon}_{t-1}$  can be used in a second regression to measure *error correction* – the rate at which equilibrium returns after a shock in  $\varepsilon$  separates  $y_t$  and  $x_t$ :

$$\Delta y_t = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1} + \Delta \beta_1 x_t + \zeta_t. \tag{3}$$

The Engle–Granger method was once a popular approach in political science (e.g. [Ostrom Jr. and Smith 1993](#); [Clarke and Stewart 1995](#); [Calderia and Zorn 1998](#)). However, the simpler single-equation GECM became the go-to method following publication of [DeBoef and Keele \(2008\)](#). The [Bårdsen \(1989\)](#) expression of a bivariate GECM is particularly popular:

$$\Delta y_t = \alpha_0 + \alpha_1^* y_{t-1} + \beta_0^* \Delta x_t + \beta_1^* x_{t-1} + \eta_t \tag{4}$$

where  $\Delta y_t$  is the differenced version of the dependent variable,  $\alpha_0$  is a constant,  $\alpha_1^*$  is the error correction coefficient,  $\beta_0^*$  is the short-term effect of  $\Delta x_t$ ,  $\beta_1^*$  is used to calculate the long-run effect (referred to as the long-run multiplier, LRM) of exogenous variable  $x_t$  as  $\frac{\beta_1^*}{-\alpha_1^*}$ , and  $\eta_t$  is a well-behaved error term. With integrated data and a single  $x$ ,  $\alpha_1^*$  tests a null of no cointegration against an alternative hypothesis that  $y_t$  and  $x_t$  are cointegrated. The test relies on non-standard “MacKinnon values” ([Ericsson and MacKinnon 2002](#)).

Equation balance is a key factor when evaluating Equation 4. The order of integration

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<sup>1</sup>We limit the deterministic features to a constant for simplicity. See Appendix for more detail.

of variables on the right-hand-side, either separately or in combination, must be the same as that of the dependent variable (Banerjee et al. 1993; Lebo and Grant 2016). A lack of balance “tells you that your model is either wrong or incomplete in a way that will prevent a meaningful interpretation of the model” (Pickup and Kellstedt 2018, p. 6). With cointegration,  $y_{t-1}$  and  $x_{t-1}$  are jointly stationary and – since  $\Delta y_t$  and  $\Delta x_t$  are each stationary – the equation is balanced. As such, each regressor can rely on a standard limiting distribution. However, if there is an integrated regressor that is not cointegrated with other variables in the equation, its coefficient cannot do so (Sims, Stock, and Watson 1990).<sup>2</sup> Thus, a standard  $t$ -test is appropriate for a regressor’s coefficient in a single equation autoregressive distributed lag model (ADL)<sup>3</sup> or GECM in the following (non-exhaustive) list of scenarios:

- A differenced  $x_t$ , whether  $x_t$  is integrated or not, though not if  $x_t$  is  $I(2)$  or higher.
- An integrated  $x_t$  in level form that is cointegrated with  $y_t$ .
- An integrated  $x_t$  alongside  $x_{t-1}$  (or other lags) making them jointly stationary.
- An integrated  $x_{1t}$  that is cointegrated with an integrated  $x_{2t}$ .
- An  $x_t$  that is stationary with little autocorrelation.

Without cointegration, Equation 4 is unbalanced since it would regress a stationary  $\Delta y$  on an  $x_{t-1}$  that is non-stationary, on its own or in combination. Then,  $\beta_1^*$  requires a non-standard distribution. In such cases, it is unclear what a long-term relationship between an  $I(0)$   $y$  and  $I(1)$   $x$ , or *vice versa*, would mean (Pickup and Kellstedt 2018). In sum, for a bivariate GECM with all  $I(1)$  data,  $\alpha_1^*$  is a test of cointegration which *must* be present for balance and for the  $\beta$ ’s to all follow a  $t$ -distribution. Next, we show the alternative hypothesis for  $\alpha_1^*$  is not straightforward with multiple independent variables.

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<sup>2</sup>Banerjee *et. al* (1993, p. 167) add: “This implies some advantage to the use of dynamic rather than static regressions, since lagging variables and including them as regressors often has the same effect as providing a co-integrated set of regressor variables.”

<sup>3</sup>The bivariate ADL is:  $y_t = \alpha + \alpha_1 y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t$ . It is mathematically equivalent to the GECM but the parameters must be interpreted differently.

### 3 Testing $\alpha_1^*$ with Multiple Independent Variables

To review, in a bivariate GECM  $\alpha_1^*$  is a cointegration test only if both  $y$  and  $x$  are  $I(1)$ . Interpretation becomes difficult without  $I(1)$  data – estimates of  $\alpha_1^*$  can depend on many factors besides the effects of independent variable. Any uncertainty in diagnosing the data creates uncertainty in what  $\alpha_1^*$  is testing.

However, even if all the data are undisputedly  $I(1)$  and even if one uses MacKinnon values, with more than a single independent variable, a significant  $\alpha_1^*$  does not necessarily indicate that *all* the variables are cointegrated nor does it mean the equation is balanced. With multiple  $x$ 's, the null hypothesis on  $\alpha_1^*$  is still that there is no cointegration but the alternative hypothesis is that cointegration exists between *at least one*  $x$  and  $y$  (Harbo et al. 1998). It is *not* that all of the  $x$ 's are cointegrated with  $y$ .

In the general case, we represent a potential cointegrating relationships between a set of variables in a vector error correction model (VECM, see Ericsson and MacKinnon 2002) as:

$$\Delta \vec{z}_t = \boldsymbol{\pi} \vec{z}_{t-1} + \boldsymbol{\Gamma} \Delta \vec{z}_t + \boldsymbol{\Phi} \vec{D}_t + \vec{\varepsilon}_t, \quad t = 1, \dots, T \quad (5)$$

where  $\vec{z}_t = (y_t, x_{1t}, \dots, x_{k-1t})'$  is a vector of  $k$  variables at time  $t$ , some of which may be cointegrated;  $\vec{D}_t$  is a vector of  $d$  deterministic variables such as a constant term and a trend; and  $\vec{\varepsilon}_t$  is a vector of  $k$  unobserved jointly normal and sequentially independent errors.<sup>4</sup> For parameters,  $\boldsymbol{\pi}$  is a  $k \times k$  matrix of coefficients on the lag of  $\vec{z}_t$ ,  $\boldsymbol{\Gamma}$  is a  $k \times k$  matrix of coefficients on the difference of  $\vec{z}_t$ ,<sup>5</sup> and  $\boldsymbol{\Phi}$  is a  $k \times d$  matrix of constant and trend coefficients.

The number of cointegrating vectors  $r$  is equal to the rank of  $\boldsymbol{\pi}$  where  $0 \leq r \leq k$ . Also,  $\boldsymbol{\pi}$  may be rewritten as  $\boldsymbol{\alpha} \boldsymbol{\beta}'$ , where  $\boldsymbol{\beta}$  is a  $k \times r$  matrix of cointegrating vectors that is of full rank, and  $\boldsymbol{\alpha}$  is a  $k \times r$  matrix of adjustment coefficients. Within this framework, Johansen's (1988; 1995) procedure determines the number of cointegrating vectors in Equation 5 based

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<sup>4</sup>For simplicity, we assumed that the maximum lag of the VECM is equal to one. See: Ericsson and MacKinnon (2002) for a more general treatment.

<sup>5</sup>Note that in this specification of the VECM,  $\text{diag}(\boldsymbol{\Gamma}) = 0$

on the rank of  $\pi$ . The key insight is that while a cointegrating vector may contain *all* the variables in a system, this is not guaranteed. Instead, it may be composed only of a *subset* of the variables and may have elements equal to zero. In fact, the possibility of multiple cointegrating vectors in the system implies that not all of them contain every single variable.<sup>6</sup> As [Banerjee et al. \(1993, 145\)](#) notes: “If  $x_t$  has  $n > 2$  components, then there may be more than one co-integrating vector  $\alpha$ ; it is possible for several equilibrium relationships to govern the joint evolution of the variables.” Even if there is only a single cointegration vector (i.e.,  $r = 1$ ), [Johansen \(1988, p. 236\)](#) says “it seems natural to test that certain variables do not enter into the cointegration vector.” Political scientists generally, but incorrectly, do not focus on the cointegrating vector(s). In their 1993 *Econometrica* article, [Stock and Watson \(1993\)](#) compare eight estimators of the cointegrating vector that allow researchers to test where cointegration is and where it is not. Assuming the independent variables are weakly exogenous and causally prior, the ECM and ADL approaches are appropriate for inference about cointegrating vector(s). Otherwise, researchers may rely on the Engle-Granger or Johansen approaches. [Enders \(2015, pp. 395-6\)](#) offers a more accessible explanation in a section called *Inference with Cointegrating Vectors*.

Applied research usually assumes weakly exogenous regressors and focuses on a single-equation GECM (i.e., only examining  $y_t$  as a dependent variable rather than the entire system  $\vec{z}_t$ ). With multiple regressors, this is:

$$\Delta y_t = \alpha_0 + \alpha_1^* y_{t-1} + \sum_{j=1}^k (\beta_{0j}^* \Delta x_{j,t} + \beta_{1j}^* x_{j,t-1}) + \varepsilon_t, \quad (6)$$

In Equation 6, the null for  $\alpha_1^* = 0$  is no cointegrating vector between  $y$  and any of the  $x$ 's. Having  $\alpha_1^* < 0$  indicates there is cointegration involving  $y$ , but, crucially, the cointegrating vector may still contain elements equal to zero. Just as cointegrating relationships in a VAR may not include all the variables in the system, in a single equation model there may be

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<sup>6</sup>The Trace and Max statistics are useful for assessing cointegrating vectors. See our Appendix for descriptions of these tests and [Box-Steffensmeier et al. \(2014, p.165\)](#) for more detail.

cointegration between  $y$  and an incomplete subset of the remaining  $x$ 's. A significant  $\alpha_1^*$  can occur in an unbalanced equation and is not evidence that all the variables are part of a cointegrating system. Some  $x$ 's may be jointly stationary and some may not be. Thus, correct interpretation of the  $\beta_1^*$  coefficients is not possible without further testing.

$$\Delta y_t = \alpha_0 + \alpha_1^* y_{t-1} + \beta_{01}^* \Delta x_{1,t} + \beta_{11}^* x_{1,t-1} + \beta_{02}^* \Delta x_{2,t} + \beta_{12}^* x_{2,t-1} + \varepsilon_t \quad (7)$$

To illustrate, consider Equation 7 when data are all  $I(1)$ ,  $y$  is cointegrated with  $x_1$ , and  $x_2$  is unrelated to both  $x_1$  and  $y$ . The terms  $y_{t-1}$  and  $x_{1,t-1}$  would be jointly stationary and likely produce a significant  $\alpha_1^*$ . However, with  $x_{2,t-1}$  included, the equation is unbalanced. Incorrect practice would use  $\alpha_1^*$  to infer that a)  $y$ ,  $x_1$ , and  $x_2$  are all part of a cointegrating system, b) the equation is balanced, and c) an asymptotically normal test-statistic applies to  $\beta_{11}^*$  and  $\beta_{12}^*$ . Since  $x_2$  is an integrated regressor, its coefficient cannot rely on the standard normal distribution. In fact, we cannot know which  $\beta_{1j}^*$ 's rely on a  $t$ -distribution and which do not unless we know which  $x$  is cointegrated. Without knowing the correct critical values, we do not know whether or not to reject the null.

## 4 Monte Carlo Analysis

Among the many applications of error correction models in political science, some use a large number of independent variables and some use very few time points.<sup>7</sup> We simulate various scenarios containing between 1 and 9 independent variables and sample sizes of 50, 100, and 200. Each regressor is simulated according to the following unit-root process:

$$x_{j,t} = x_{j,t-1} + \varepsilon_{j,t}, \quad \varepsilon_{j,t} \sim N(0, 1), \quad j = 1, \dots, k; \quad t = 1, \dots, T. \quad (8)$$

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<sup>7</sup>For example, [Volscho and Kelly \(2012\)](#) use around 10 independent variables (depending on the model specification) and [Ura and Wohlfarth \(2010\)](#) have  $T = 29$ .

Then, we generate  $y_t$  such that:

$$\zeta_t = 0.6 * \zeta_{t-1} + \eta_t, \quad \eta_t \sim N(0, 1) \quad (9)$$

$$y_t = x_{1,t} + \zeta_t \quad (10)$$

This DGP creates a cointegrating relationship between  $y$  and  $x_1$ , since both are  $I(1)$  and their difference is AR(1) with  $\gamma = 0.6$ .<sup>8</sup> The remaining independent variables  $x_2, \dots, x_9$  are each  $I(1)$  but unrelated to other variables. For each scenario, we generate 1000 simulated datasets and estimate GECMs of the form:

$$\Delta y_t = \alpha_0 + \alpha_1^* y_{t-1} + \sum_{j=1}^k (\beta_{0j}^* \Delta x_{j,t} + \beta_{1j}^* x_{j,t-1}) + \varepsilon_t, \quad (11)$$

with  $k$  varying from 1 through 9. Starting with a balanced GECM where the only independent variable is  $x_1$ , we incrementally add unrelated  $I(1)$  regressors until we have included all of  $x_1$  through  $x_9$ . Moving from left to right within each panel of Figure 1 shows the proportion of times  $\alpha_1^*$  surpasses MacKinnon's critical values ( $p < .05$ ) as unrelated regressors are added alongside the cointegrated  $x_1$ .

In practice, it is clear that  $\alpha_1^*$  tests whether cointegration is present, *not* whether all the variables are jointly cointegrated. For example, with eight unrelated  $I(1)$  regressors,  $\alpha_1^*$  identifies that cointegration is present in 100% and 82% of simulations for  $T = 200$  and  $T = 100$ , respectively.<sup>9</sup> Additional unrelated  $I(1)$  regressors modestly reduce the frequency with which  $\alpha_1^*$  reaches significance in shorter time series but, usually,  $\alpha_1^*$  does not alert the researcher that any particular  $x$  is not part of the equilibrium relationship with  $y$ . If a significant  $\alpha_1^*$  indicated that *all* of the variables are a part of a cointegrating system, it would have to cease being significant once the model contained an unrelated  $I(1)$  regressor.

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<sup>8</sup>When estimating a bivariate GECM, this results in a cointegrating relationship with  $\alpha_1^* \approx -0.4$ . Alternatively, we also simulated a DGP that directly implements the GECM specified in Equation (1) for  $y$  and  $x_1$ . For example, setting  $\alpha_0 = 1$ ,  $\alpha_1^* = -0.4$ ,  $\beta_0^* = 0.5$ , and  $\beta_1^* = 0.5$  yields results that are almost identical to the ones discussed below. For further details see online appendix.

<sup>9</sup>These problems are exacerbated if the assumption of weak exogeneity does not hold. See Appendix.



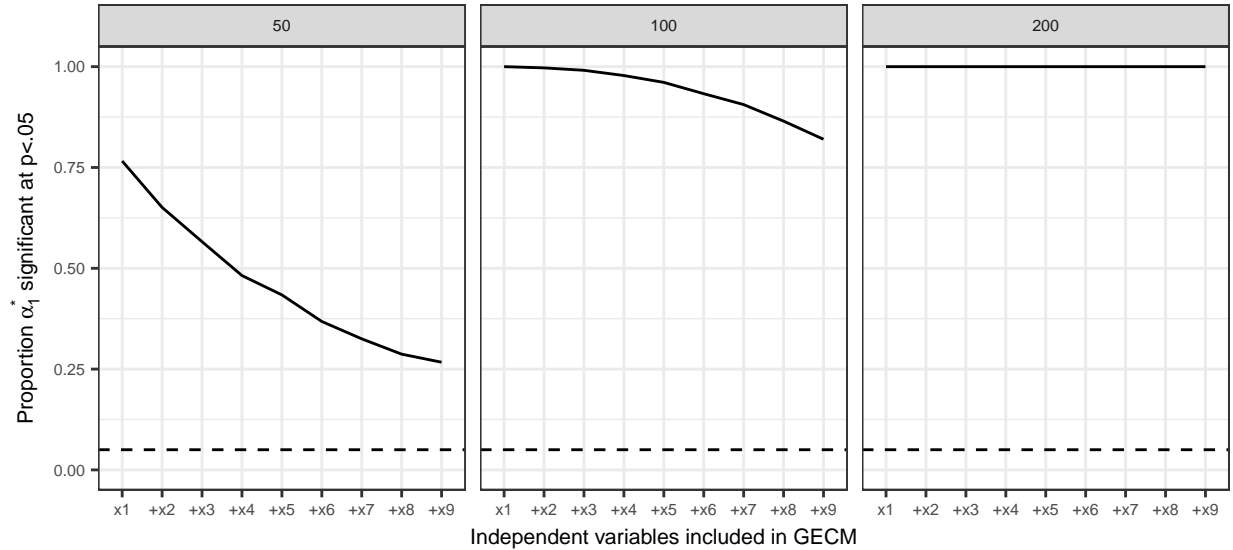


Figure 1: The consequences of GECMs with unbalanced equations. Adding unrelated  $I(1)$  regressors does not sufficiently diminish the statistical significance of  $\alpha_1^*$ .

Problems extend to long-run multipliers, calculated as  $\frac{\beta_{1j}^*}{-\alpha_1^*}$ . Conditional on  $\alpha_1^*$  surpassing MacKinnon critical values, Figure 2 displays the average proportion of times each LRM for the unrelated independent variables  $x_2$  through  $x_9$  are significantly different from zero. Rejecting the null conditional on  $\alpha_1^*$ , we observe inflated false positive rates on all unrelated  $x$ 's that are included in the model. Instead of an appropriate rejection rate of 5% (horizontal reference), unrelated regressors are statistically significant far too often. Additional  $x$ 's also move  $\alpha_1^*$  further from 0 which might lead a researcher to erroneously describe a faster error correction rate.<sup>10</sup>

Using the incorrect alternative hypothesis for  $\alpha_1^*$  with multiple  $I(1)$   $x$ 's and rejecting the null without further investigation means interpreting  $\beta$ 's without knowing where one might be breaking the standard of zero-mean non-integrated regressors needed for trustworthy tests using a standard  $t$ -distribution. Before being able to consider the results of a GECM for substantive interpretation, applied researchers must make sure that the only  $I(1)$   $x$ 's included in the model are those that are indeed part of the cointegrating system. Again, [Stock and Watson \(1993\)](#) compare eight procedures to examine the cointegrating vector(s).

<sup>10</sup>See online appendix for additional results.

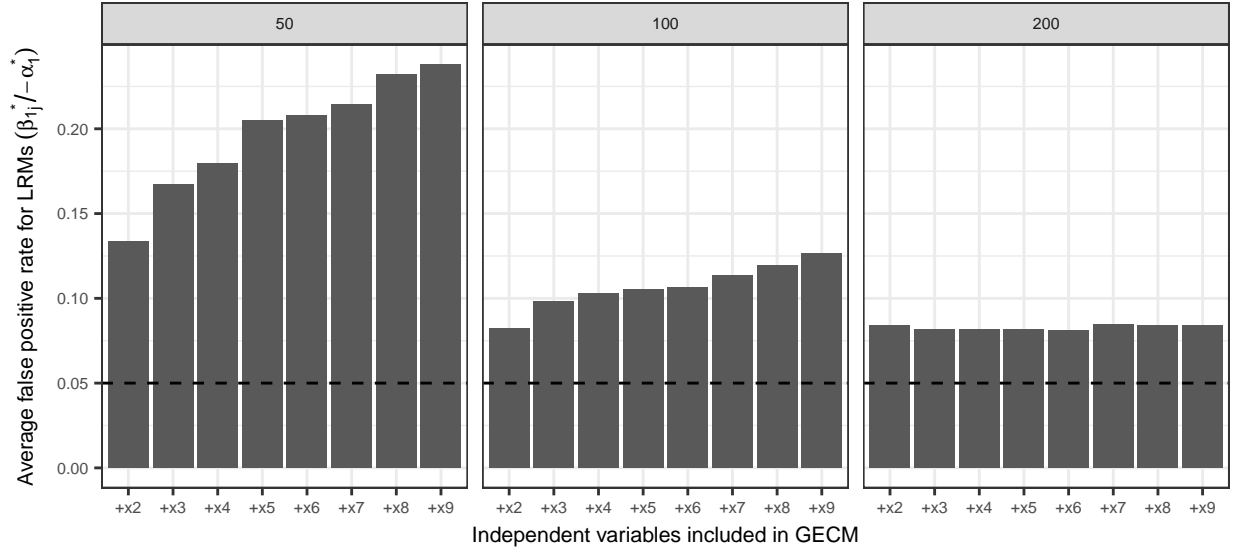


Figure 2: The consequences of GECMs with unbalanced equations. We observe inflated false positives on long run multipliers for unrelated regressors. The horizontal line indicates an acceptable significance rate of 0.05.

Researchers need to be familiar with and apply some of these tests when attempting to make inferences with more than a single exogenous variable. The following section discusses examples where this practice has not been followed.

## 5 Examples: Kelly and Enns (2010) and Volscho and Kelly (2012)

Many GECM analyses in political science rely on  $\alpha_1^*$  to judge cointegration and error correction between multiple variables simultaneously. For example, Enns et al. (2016, 4) apply MacKinnon values to data from Kelly and Enns (2010) and claim to “find clear evidence of cointegration” between *Liberal Policy Mood* and various sets of independent variables in their GECMs.

Even assuming the data are  $I(1)$ , Enns et al. (2016) use a significant  $\alpha_1^*$  to conclude that *all* of the series are cointegrated. Relying on only  $\alpha_1^*$ , it is unknown if the equation is balanced. For example, Kelly and Enns’s (2010) Model 2 in Table 1 reports coefficients for policy liberalism and income inequality which may or may not be significant; the correct

limiting distributions cannot be known without more testing.<sup>11</sup>

Volscho and Kelly (2012) estimate the determinants of Income for the Top 1% using:<sup>12</sup>

$$\begin{aligned}
\Delta Top1\%_t = & \alpha_0 + \alpha_1^* Top1\%_{t-1} + \beta_{1(CD)}^* \%CongDems_{t-1} + \beta_{1(DG)}^* \%DividedGovt_{t-1} \\
& + \beta_{1(UM)}^* \%UnionMembership_{t-1} + \beta_{1(TMT)}^* \%TopMarginalTax_{t-1} \\
& + \beta_{0(CGT)}^* \Delta CapitalGainsTaxRate_t + \beta_{1(CGT)}^* CapitalGainsTaxRate_{t-1} \\
& + \beta_{1(3MTB)}^* 3MonthTBill_{t-1} + \beta_{0(TO)}^* \Delta TradeOpenness_t + \beta_{1(LogRGDP)}^* LogRGDP_{t-1} \\
& + \beta_{0(RealS\&P)}^* \Delta RealS\&P500Index_t + \beta_{1(RealS\&P)}^* RealS\&P500Index_{t-1} \\
& + \beta_{1(SHPI)}^* ShillerHPI_{t-1} + \epsilon_t
\end{aligned} \tag{12}$$

Without multiple lags of the  $x$ 's, the integrated right-hand-side variables must all be mutually cointegrated for equation balance to hold and for the  $\beta_1$ 's to rely on the  $t$ -distribution. How should we test cointegration here? Volscho and Kelly (2012) use a significant  $\alpha_1^*$  as evidence that *all*  $I(1)$  regressors are cointegrated.

Enns and Wlezien (2017) claim that Volscho and Kelly's equation is balanced so that  $\beta_{1(UM)}^*$ ,  $\beta_{1(TMT)}^*$ ,  $\beta_{1(CGT)}^*$ ,  $\beta_{1(3MTB)}^*$ ,  $\beta_{1(LogRGDP)}^*$ , and  $\beta_{1(RealS\&P)}^*$  can be evaluated using a standard normal distribution. The reasoning seems to be that, since a significant  $\alpha_1^*$  indicates all the unit root  $x$ 's are in a cointegrating system, and since stationary variables also rely on a  $t$ -test, everything on the right-hand-side of the equation must be stationary and can rely on a  $t$ -test. This rationale makes stationarity concerns inconsequential and is a step away from calls for more careful analyses.

What if, like our simulations,  $Top1\%Share$  is in fact cointegrated with some but not all of the  $I(1)$   $x$ 's? If so, some  $\beta$ 's are trustworthy and others are not. How can we tell which is which? To experiment, we swapped out  $\Delta CapitalGainsTaxRate_t$  and  $3MonthTBill_{t-1}$  in favor of  $\Delta Unemployment_t$  and  $Unemployment_{t-1}$ . Volscho and Kelly (2012) acknowledge

<sup>11</sup>See Lebo and Kraft (2017) for further examination of issues with Kelly and Enns (2010).

<sup>12</sup>Beyond adding independent variables, Equation 12 is not a straightforward expansion of a GECM as several components such as  $\Delta \% CongDems_t$  have been left out.

the latter two are not predictors of *Top1%Share* and omit them from their preferred model. The new model's results do not alert us that the cointegrating system has an intruder. In fact,  $\alpha_1^*$  moves farther from 0, from -0.648 to -0.759, remains significant, and surpasses the MacKinnon critical value.<sup>13</sup> A better approach within the GECM framework would do subsequent testing to piece together where cointegration is and where it is not. We conclude with a brief overview of best practices.

## 6 Discussion

In a single equation model with  $I(1)$  data, a significant  $\alpha_1^*$  indicates that at least one regressor is cointegrated with the dependent variable. It does not test whether multiple  $x$ 's are all cointegrated with the dependent variable. Without understanding the alternative hypothesis, we can mistakenly think an equation is balanced and perhaps use the wrong limiting distribution and critical value to incorrectly reject a true null hypothesis. Many extant studies in political science run afoul of what we now know to be good practice. As [Banerjee et al. \(1993, p.192\)](#) point out, "The moral of the econometricians' story is the need to keep track of the orders of integration on both sides of the regression equation." In light of our findings in this paper, we recommend prior studies be read cautiously and reexamined.

How can practitioners make reliable inferences using the GECM? First, be less ambitious with short data sets. [Keele, Linn, and Webb \(2016\)](#) suggest one regressor for every 10 observations as a rule of thumb. Second, demonstrate robustness by trying models with different assumptions regarding the underlying univariate processes. Third, if one assumes that  $y$  and multiple  $x$ 's are  $I(1)$ , take great care to properly identify the cointegrating system. One possibility is to apply the Engle-Granger (1987) two-step cointegration process iteratively by adding individual regressors in order to sort which variables are cointegrated and which are not. Alternatively, Johansen's (1988; 1995) procedure allows for direct inference on the cointegrating vector to identify variables included in the equilibrium relationship.<sup>14</sup> Also,

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<sup>13</sup>See online appendix for additional results.

<sup>14</sup>See [Enders \(2015\)](#); [Box-Steffensmeier et al. \(2014\)](#) as well as the online appendix for more information.

Stock and Watson (1993) estimate the cointegrating vector using dynamic OLS and parse out which variables are cointegrated. Ultimately, researchers must include  $I(1)$   $x$ 's as regressors only when they are part of the cointegrating system or otherwise mutually stationary with another regressor.

Fourth, when using the GECM, rely on long-run multipliers instead of  $\alpha_1^*$  (Banerjee et al. 1993, Chapter 2; DeBoef and Keele 2008). Finally, consider new methods that forego the knife-edged classification decision between  $I(0)$  and  $I(1)$  (e.g. Lebo and Young (2009); Lebo and Norpoth (2011)) or that entirely avoid univariate identification such as the bounds procedure introduced by Webb, Linn, and Lebo (2019, 2020).

Overall, researchers should be careful when positing about more than two variables as being part of a cointegrating “system” (e.g., Ramirez 2009; Ura and Wohlfarth 2010; Ura 2014; Enns 2014). Rather than quickly interpreting one parameter of a fully specified GECM as evidence for joint cointegration of all  $I(1)$  regressors, it is helpful to examine cointegrating vectors one independent variable at a time. Equation balance is a useful concept for understanding when hypothesis tests follow standard limiting distributions but exclusively relying on a single parameter in the GECM is insufficient to assess balance and make credible inferences.

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